

## ON PRE $R_g$ -CONTINUOUS FUNCTIONS IN TOPOLOGY

Govindappa Navalagi<sup>1</sup> and Sujata Mookanagoudar<sup>2\*</sup>

<sup>1</sup>Department of Mathematics, KIT Tiptur-572202, Karnataka, India. Email: [gnavalagi@hotmail.com](mailto:gnavalagi@hotmail.com)

<sup>2\*</sup>Department of Mathematics, Government First Grade College, Haliyal- 581329, Karnataka, India.  
Email: [suja\\_goudar82@rediffmail.com](mailto:suja_goudar82@rediffmail.com)

**\*Corresponding Author: -**

Email: [suja\\_goudar82@rediffmail.com](mailto:suja_goudar82@rediffmail.com)

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### **Abstract: -**

*In this paper we define and study the concept of pre- $R_g$ -continuous functions using preopen sets due to Mashhour et.al (1982) and  $rg$ -open sets due to N. Palaniappan (1993). Also, we study the pre- $g$ -regular spaces.*

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## 1. INTRODUCTION

In 1982, A. S. Mashhour et.al [8] has defined and studied the concepts of preopen sets and precontinuous functions in topology. For the first time the concept of generalized closed sets was considered by Levine in 1970 [7]. Later many generalized closed sets have been defined and studied by various Authors in the literature. In 1993, 1997 and 1998 respectively, N. Palaniappan [13], T. Noiri [12] et.al have defined and studied the concepts of rg-closed sets, rg-continuous functions, rg-irresolute functions, gpr-closed sets, gp-closed sets and gp-closed functions in topology. In this paper we define and study the concept of pre-Rg-continuous functions using preopen sets due to Mashhour et.al [8] and rg-open sets due to N. Palaniappan [13].

## 2. Preliminaries

In what follows, spaces  $X$  and  $Y$  are always topological spaces.  $Cl(A)$  and  $Int(A)$  designate the closure and the interior of  $A$  which is a subset of  $X$ . A subset  $A$  is said to be regular open (resp. regular closed) if  $A = Int(Cl(A))$  (resp.  $A = Cl(Int(A))$ ).

The following definitions and results are useful in sequel.

**Definition 2.1:** A subset  $A$  of a space  $X$  is called:

- (i) Preopen [8] if  $A \subset Int(Cl(A))$ ,
- (ii) Preclosed [5] if  $Cl(Int(A)) \subset A$ .

The collection of all preopen (resp. preclosed) sets of space  $X$  will be denoted by  $PO(X)$  (resp.  $PF(X)$ ).

**Definition 2.2:** A subset  $A$  of  $X$  is called preregular [11] if it is both preopen and preclosed set.

The family of all preregular sets of  $X$  is denoted by  $PR(X)$ . The complement of a preregular set is also preregular.

**Definition 2.3:** Let  $A$  be a subset of a space  $X$  then

- i. The intersection of all preclosed sets containing  $A$  is called the pre-closure [5] of  $A$  and is denoted by  $pCl(A)$ .
- ii. The intersection of all regular closed sets containing  $A$  is called the  $r$ -closure [3] of  $A$  and is denoted by  $rCl(A)$ .
- iii. The union of all preopen sets contained in  $A$  is called the preInterior [9] of  $A$  and is denoted by  $pInt(A)$ .
- iv. The union of all regular open sets contained in  $A$  is called the  $r$ -Interior[3] of  $A$  and is denoted by  $rInt(A)$ .

**Definition 2.4:** A function  $f: X \rightarrow Y$  is said to be precontinuous [8] if  $f^{-1}(V)$  is preopen in  $X$  for each open set  $V$  of  $Y$ .

**Definition 2.5:** A subset  $A$  of a space  $X$  is said to be:

- i. a generalized closed set (briefly  $g$  – closed) [7] if  $Cl(A) \subset U$  whenever  $A \subseteq U$  and  $U$  is open in  $X$ .
- ii. a generalized preclosed set (briefly  $gp$  – closed) [12] if  $pCl(A) \subset U$  whenever  $A \subseteq U$  and  $U$  is open in  $X$ .
- iii. a  $r$ -generalized closed set (briefly  $rg$  – closed) [13] if  $rCl(A) \subset U$  whenever  $A \subseteq U$  and  $U$  is open in  $X$ .
- iv. a generalized pre-regular closed set (briefly  $gore$  – closed) [6] if  $pCl(A) \subset U$  whenever  $A \subseteq U$  and  $U$  is regular open in  $X$ .

The complement of a  $g$ -closed (resp.  $gp$ -closed,  $rg$ -closed and  $gpr$ -closed) set of a space  $X$  is called  $g$ -open (resp.  $gp$ -open,  $rg$ -open and  $gpr$ -open) set in  $X$ .

**Definition 2.6:** A function  $f: X \rightarrow Y$  is said to be:

- (i)  $g$ -continuous [4] if  $f^{-1}(V)$  is  $g$ -closed in  $X$  for each closed set  $V$  of  $Y$ .
- (ii)  $rg$ -continuous [13] if  $f^{-1}(V)$  is  $rg$ -closed in  $X$  for each closed set  $V$  of  $Y$ .
- (iii)  $rg$ -irresolute [13] if  $f^{-1}(V)$  is  $rg$ -open in  $X$  for each  $rg$ -open set  $V$  of  $Y$ .
- (iv)  $gp$ -continuous [2] if  $f^{-1}(V)$  is  $gp$ -closed in  $X$  for each closed set  $V$  of  $Y$ .
- (v)  $gp$ -irresolute [2] if  $f^{-1}(V)$  is  $gp$ -open in  $X$  for each  $gp$ -open set  $V$  of  $Y$ .

**Definition 2.7:** A topological space  $X$  is said to be:

- (i)  $p$ -regular [5] if for each closed set  $A$  and each point  $x \in X - A$  there exist preopen sets  $U, V$  such that  $x \in U, A \subset V$  and  $U \cap V = \emptyset$ .
- (ii)  $g$ -regular [10] if for each closed set  $A$  and each point  $x \in X - A$  there exist generalized open sets  $U, V$  such that  $x \in U, A \subset V$  and  $U \cap V = \emptyset$ .
- (iii) pre-connected [14] if  $X$  cannot be written as disjoint union of two non- empty preopen sets.
- (iv)  $rg$ -connected [1] if  $X$  cannot be written as disjoint union of two non-empty  $rg$ -open sets.

## 3. Properties of pre-Rg-continuous functions

In this section we define the following.

**Definition 3.1:** A function  $f: X \rightarrow Y$  is called pre-Rg-continuous if the inverse image of each rg-open set of  $Y$  is preopen in  $X$ .

We have the following implications:

- (i) Every strongly rg-continuous function is pre-Rg-continuous.
- (ii) Every pre-Rg-continuous function is almost precontinuous.

We prove the following.

**Theorem 3.2:** Let  $f: X \rightarrow Y$  be a single valued function, where  $X$  and  $Y$  are topological spaces, then the following are equivalent

- (i) The function  $f$  is pre-Rg-continuous.
- (ii) For each point  $x \in X$  and each rg-open set  $V$  in  $Y$  with  $f(x) \in V$ , there is a preopen set  $U$  in  $X$  such that  $x \in U$ ,  $f(U) \subset V$ .

**Proof :** (i)  $\Rightarrow$  (ii):

Let  $f(x) \in V$  and  $V \subset Y$  an rg-open set then  $x \in f^{-1}(V) \in PO(X)$  as  $f$  is pre-Rg-continuous. Let  $U = f^{-1}(V)$ , then  $x \in U$  and  $f(U) \subset V$ . Conversely, Let  $V$  be rg-open in  $Y$  and  $x \in f^{-1}(V)$  then  $f(x) \in V$ , there exists a  $U_p \in PO(X)$  such that  $p \in U_p$  and  $f(U_p) \subset V$ . Then  $x \in U_x \subset f^{-1}(V)$  and  $f^{-1}(V) = \bigcup U_x \in PO(X)$ . This implies  $f$  is pre-Rg-continuous. We define the following.

**Definition 3.3 :** A function  $f: X \rightarrow Y$  is called gp-rg-continuous if the inverse image of each rg-open subset of  $Y$  is gp-open in  $X$ .

Clearly, Every pre-rg-continuous function is gp-rg-continuous function.

**Theorem 3.4:** Let  $f: X \rightarrow Y$  be pre-Rg-continuous then  $f$  is gp-rg-continuous.

**Proof:** Let  $V \subset Y$  be rg-open. Then  $f^{-1}(V)$  is preopen in  $X$ , since  $f$  is pre-Rg-continuous. Since every preopen set is gp-open set, then  $f^{-1}(V)$  is gp-open set in  $X$ . Thus,  $f$  is gp-rg-continuous.

We define the following.

**Definition 3.5:** A function  $f: X \rightarrow Y$  is called gp-gpr-continuous if the inverse image of each gpr-open subset of  $Y$  is gp-open in  $X$ .

Clearly, Every gp-rg-continuous function is gpr-irresolute.

**Definition 3.6:** A function  $f: X \rightarrow Y$  is called pre-strongly-gpr-continuous if the inverse image of each gpr-open set of  $Y$  is preopen in  $X$ .

**Note 3.7:**

- (i) Every strongly rg-continuous function is pre-Rg-continuous.
- (ii) Every pre-Rg-continuous function is almost precontinuous.

**Theorem 3.8:** Let  $f: X \rightarrow Y$  be a function. Then the following are equivalent.

- (i)  $f$  is pre-Rg-continuous.
- (ii) The inverse image of each rg-open set in  $Y$  is preopen in  $X$ .
- (iii) The inverse image of each rg-closed set in  $Y$  is preclosed in  $X$ .

**Proof :** (i)  $\Rightarrow$  (ii) :

Let  $G$  be any rg-open set of  $Y$ . Then,  $Y - G$  is rg-closed in  $Y$ . By the assumption of (i),  $f^{-1}(Y - G)$  is preclosed set in  $X$ . But  $f^{-1}(Y - G) = X - f^{-1}(G)$  which implies that  $X - f^{-1}(G)$  is preclosed in  $X$ . Therefore  $f^{-1}(G)$  is preopen in  $X$ .

(ii)  $\Rightarrow$  (iii) : Obvious.

(iii)  $\Rightarrow$  (i) : Obvious.

**Theorem 3.9:** Let  $f: X \rightarrow Y$  is pre-Rg-continuous surjection and  $X$  is preconnected then  $Y$  is rg-connected.

**Proof:** Suppose  $Y$  is not rg-connected. Let  $Y = A \cup B$ , where  $A$  and  $B$  are disjoint non-empty rg-open sets in  $Y$ . Since  $f$  is pre-Rg-continuous and onto,  $X = f^{-1}(A) \cup f^{-1}(B)$ , where  $f^{-1}(A)$  and  $f^{-1}(B)$  are disjoint non-empty and preopen sets in  $X$ . This contradicts the fact that  $X$  is preconnected. Hence  $Y$  is rg-connected.

**Theorem 3.10:** Let  $f: X \rightarrow Y$  is gp-rg-continuous surjection and  $X$  is gp-connected then  $Y$  is rg-connected.

**Proof:** Suppose  $Y$  is not rg-connected. Let  $Y = A \cup B$ , where  $A$  and  $B$  are disjoint non-empty rg-open sets in  $Y$ . Since  $f$  is gp-rg-continuous and onto,  $X = f^{-1}(A) \cup f^{-1}(B)$ , where  $f^{-1}(A)$  and  $f^{-1}(B)$  are disjoint non-empty and gp-open sets in  $X$ . This contradicts the fact that  $X$  is gp-connected. Hence  $Y$  is rg-connected.

**Theorem 3.11:** Let  $f : X \rightarrow Y$  is pre-strongly-gpr-continuous surjection and  $X$  is preconnected then  $Y$  is gpr-connected.

**Proof:** Suppose  $Y$  is not gpr-connected. Let  $Y = A \cup B$ , where  $A$  and  $B$  are disjoint non-empty gpr-open sets in  $Y$ . Since  $f$  is pre-strongly-gpr-continuous and Onto,  $X = f^{-1}(A) \cup f^{-1}(B)$ , where  $f^{-1}(A)$  and  $f^{-1}(B)$  are disjoint non-empty and preopen sets in  $X$ . This contradicts the fact that  $Y$  is preconnected. Hence  $Y$  is gpr-connected.

**Theorem 3.12:** Let  $f : X \rightarrow Y$  is strongly rg-continuous surjection and  $X$  is connected then  $Y$  is rg-connected.

**Proof:** Suppose  $Y$  is not rg-connected. Let  $Y = A \cup B$ , where  $A$  and  $B$  are disjoint non-empty rg-open sets in  $Y$ . Since  $f$  is strongly-rg-continuous and Onto,  $X = f^{-1}(A) \cup f^{-1}(B)$ , where  $f^{-1}(A)$  and  $f^{-1}(B)$  are disjoint non-empty and open sets in  $X$ . This contradicts the fact that  $X$  is connected. Hence  $Y$  is rg-connected.

We define the following.

**Definition 3.13:** A function  $f : X \rightarrow Y$  is said to be contra-pre-Rg-continuous if the inverse image of each rg-open set of  $Y$  is preclosed in  $X$ .

**Definition 3.14:** A function  $f : X \rightarrow Y$  is said to be contra strongly-rg-continuous if the inverse image of each rg-open set of  $Y$  is closed in  $X$ .

Since every open set is rg-open set and every closed set is preclosed set and hence we have the following implications.

- (i) Every contra strongly rg-continuous function is contra-pre-Rg-continuous.
- (ii) Every contra-pre-Rg-continuous function is contra-precontinuous.

**Theorem 3.15 :** The following are equivalent for a function  $f : X \rightarrow Y$  :

- (i)  $f$  is contra-pre-Rg-continuous.
- (ii) For each rg-closed subset  $F$  of  $Y$ ,  $f^{-1}(F) \in PO(X)$ ,
- (iii) For each  $x \in X$  and rg-closed subset of  $Y$  containing  $f(x)$ , there exists preopen set  $U$  in  $X$  containing point  $x$  such that  $f(U) \subset F$ .

**Proof :** (i)  $\Rightarrow$  (ii) :

Let  $F$  be any rg-closed subset of  $Y$ . Then,  $Y - F$  be any rg-open subset of  $Y$ . Since by (i),  $f^{-1}(Y - F) = X - f^{-1}(F)$  is preclosed set in  $X$ . Hence,  $f^{-1}(F)$  is preopen set in  $X$ . Thus, (ii) holds.

(ii)  $\Rightarrow$  (i) : Obvious.

(ii)  $\Rightarrow$  (iii) : Obvious.

(iii)  $\Rightarrow$  (ii) :

Let  $F$  be any rg-closed subset of  $Y$  and  $x \in f^{-1}(F)$ . Then  $f(x) \in F$  and there exists a preopen set  $U_x$  in  $X$  containing point  $x$  such that  $f(U_x) \subset F$ . Therefore, we obtain  $f^{-1}(F) = \bigcup \{ U_x \mid x \in f^{-1}(F) \} \in PO(X)$ . This shows that (ii) holds. We define the following.

**Definition 3.16 :** A topological space  $X$  is said to be pre-rg-regular if for each rg-closed set  $A$  and each point  $x \in X - A$  there exists preopen sets  $U, V$  such that  $x \in U, A \subset V$  and  $U \cap V = \emptyset$ .

**Definition 3.17 :** A topological space  $X$  is said to be pre-g-regular if for each g-closed set  $A$  and each point  $x \in X - A$  there exists preopen sets  $U, V$  such that  $x \in U, A \subset V$  and  $U \cap V = \emptyset$ .

Since every closed set is g-closed set and every g-closed set is rg-closed set and every closed set is rg-closed set. Thus we have the following implications.

- (i) Every pre-Rg-regular space is p-regular.
- (ii) Every pre-g-regular space is p-regular.
- (iii) Every pre-Rg-regular space is pre-g-regular.

We state the following.

**Theorem 3.18:** For a topological space  $X$  the following hold:

- (i)  $X$  is a pre-rg-regular.
- (ii) For each  $x \in X$  and each rg-open set  $U$  of  $X$  containing  $x$ , there exists  $V \in PO(X)$  such that  $x \in V \subset pCl(V) \subset U$ .
- (iii) For each rg-closed set  $F$  of  $X$ ,  $\bigcap \{ pCl(V) \mid F \subset V \in PO(X) \} = F$ .

**Theorem 3.19:** For a topological space  $X$  the following hold:

- (i)  $X$  is a pre-g-regular.
- (ii) For each  $x \in X$  and each g-open set  $U$  of  $X$  containing  $x$ , there exists  $V \in PO(X)$  such that  $x \in V \subset pCl(V) \subset U$ .
- (iii) For each g-closed set  $F$  of  $X$ ,  $\bigcap \{ pCl(V) \mid F \subset V \in PO(X) \} = F$ .

**Theorem 3.20:** If a function  $f : X \rightarrow Y$  is said to be contra-pre-rg-continuous and  $Y$  is pre-rg-regular, then  $f$  is pre-rg-continuous.

**Proof :** Let  $x$  be any arbitrary point of  $X$  and  $V$  be an rg-open set of  $Y$  containing  $f(x)$ . Since  $Y$  is pre-rg-regular, there exists an pre-open set  $w$  in  $Y$  containing  $f(x)$  such that  $pCl(w) \subset V$ . Since  $f$  is contra-pre-rg-continuous, so by Theorem 3.15 there exists preopen set  $U$  in  $X$  containing point  $x$  such that  $f(U) \subset pCl(w)$ . Then  $f(U) \subset pCl(w) \subset V$ . Hence  $f$  is pre-

rg-continuous.

We define the following

**Definition 3.21 :** A function  $f : X \rightarrow Y$  is said to be perfectly pre-Rg-continuous if  $f^{-1}(V)$  is pre-regular set in  $X$  for each rg-open set  $V$  in  $Y$ .

**Lemma 3.22 :** Let  $f : X \rightarrow Y$  be a function. Then,

- (i) If  $f$  is perfectly pre-Rg-continuous, then  $f$  is pre-rg-continuous.
- (ii) If  $f$  is perfectly pre-Rg-continuous, then  $f$  is Contra pre-Rg-continuous.

**Proof:**

- (i) Let  $U$  be an rg-open set in  $Y$ , since  $f$  is perfectly pre-Rg-continuous, then  $f^{-1}(U)$  is pre-regular in  $X$ . But every pre-regular set is pre-open, then  $f^{-1}(U)$  is pre-open in  $X$ . Hence  $f$  is perfectly pre-Rg-continuous.
- (ii) Let  $U$  be an rg-open set in  $Y$ , Since  $f$  is perfectly pre-Rg-continuous, then  $f^{-1}(U)$  is pre-regular in  $X$ . But every pre-regular set is pre-closed, then  $f^{-1}(U)$  is preclosed in  $X$ . Hence  $f$  is Contra pre-Rg-continuous.

**Theorem 3.23:** Let  $f : X \rightarrow Y$  be a function. Then the following are equivalent.

- (i) If  $f$  is perfectly pre-Rg-continuous.
- (ii) The inverse image of every rg-open set in  $Y$  is both preopen and preclosed in  $X$ .
- (iii) The inverse image of every rg-closed set in  $Y$  is both preopen and preclosed in  $X$ .

**Proof :**

- (i)  $\Rightarrow$  (ii) : Clearly from the definition.
- (ii)  $\Rightarrow$  (iii) : Let  $F$  be any rg-closed set in  $Y$ . Then,  $(Y-F)$  is rg-open in  $Y$ . Hence by assumption  $f^{-1}(Y-F)$  is both pre-open and pre-closed in  $X$ .
- (iii)  $\Rightarrow$  (i) : Let  $G$  be any rg-open set in  $Y$ . Then,  $(Y-G)$  is rg-closed in  $Y$ . Hence by the condition  $f^{-1}(Y-G)$  is both pre-open and pre-closed in  $X$  which implies that  $f^{-1}(G)$  is both preopen and preclosed in  $X$ . Hence  $f$  is perfectly pre-Rg-continuous.

**Theorem 3.25 :** Let  $f : X \rightarrow Y$  and  $g : X \rightarrow Y$  be two functions. Then

- (i)  $g \circ f$  is pre-Rg-continuous, if  $g$  is gp-rg-continuous and  $f$  is pre-Rg-continuous.
- (ii)  $g \circ f$  is pre-Rg-continuous, if  $g$  is gp-rg-continuous and  $f$  is pre-strongly-gp-continuous.
- (iii)  $g \circ f$  is pre-Rg-continuous, if  $g$  is strongly-rg-continuous and  $f$  is pre-continuous.

**Proof :**

- i. Let  $V$  be arg-open set in  $Z$ . As  $g : Y \rightarrow Z$  is rg-irresolute,  $g^{-1}(V)$  is rg-open in  $Y$ . Again,  $f$  is pre-Rg-continuous and  $g^{-1}(V)$  is rg-open in  $Y$ . Then  $f^{-1}(g^{-1}(V)) = (g \circ f)^{-1}(V)$  Preopen set in  $X$ . Therefore  $g \circ f$  is pre-Rg-continuous.
- ii. Let  $V$  be arg-open set in  $Z$ . As  $g : Y \rightarrow Z$  is gp-rg-continuous,  $g^{-1}(V)$  is gp-open in  $Y$ . Again,  $f$  is pre-strongly-gp-continuous and  $g^{-1}(V)$  is gp-open in  $Y$ . Then  $f^{-1}(g^{-1}(V)) = (g \circ f)^{-1}(V)$  is Preopen set in  $X$ . Therefore  $g \circ f$  is pre-Rg-continuous.
- iii. Let  $V$  be arg-open set in  $Z$ . As  $g : Y \rightarrow Z$  is strongly-rg-continuous,  $g^{-1}(V)$  is open in  $Y$ . Again,  $f$  is pre-continuous and  $g^{-1}(V)$  is open in  $Y$ . Then  $f^{-1}(g^{-1}(V)) = (g \circ f)^{-1}(V)$  Preopen set in  $X$ . Therefore  $g \circ f$  is pre-Rg-continuous.

**Theorem 3.25 :** Let  $f : X \rightarrow Y$  and  $g : X \rightarrow Y$  be two functions. Then

- (i)  $g \circ f$  is precontinuous, if  $f$  is pre-Rg-continuous and  $g$  is rg-continuous.
- (ii)  $g \circ f$  is precontinuous, if  $f$  is pre-strongly-rg-continuous and  $g$  is rg-continuous.
- (iii)  $g \circ f$  is continuous, if  $f$  is strongly-rg-continuous and  $g$  is rg-continuous.

**Proof :**

- i. Let  $V \subset Z$  be an arbitrary open set. Since  $g$  is rg-continuous,  $g^{-1}(V)$  is rg-open set in  $Y$ . Again, as  $g^{-1}(V)$  is rg-open set in  $Y$  and  $f$  is pre-Rg-continuous function,  $f^{-1}(g^{-1}(V)) = (g \circ f)^{-1}(V)$  is preopen set in  $X$ . This shows that  $g \circ f$  is precontinuous function.
- ii. Let  $V \subset Z$  be an arbitrary open set. Since  $g$  is rg-continuous function,  $g^{-1}(V)$  is rg-open set in  $Y$ . Again, as  $g^{-1}(V)$  is rg-open set in  $Y$  and  $f$  is pre-strongly-rg-continuous function,  $f^{-1}(g^{-1}(V)) = (g \circ f)^{-1}(V)$  is preopen set in  $X$ . This shows that  $g \circ f$  is precontinuous function.
- iii. Let  $V \subset Z$  be an arbitrary open set. Since  $g$  is rg-continuous function,  $g^{-1}(V)$  is rg-open set in  $Y$ . Again, as  $g^{-1}(V)$  is rg-open set in  $Y$  and  $f$  is strongly-rg-continuous function,  $f^{-1}(g^{-1}(V)) = (g \circ f)^{-1}(V)$  is open set in  $X$ . This shows that  $g \circ f$  is continuous function.

**Lemma 3.26 :** If  $f : X \rightarrow Y$  is strongly preopen surjective and  $g : Y \rightarrow Z$  is a function such that  $g \circ f : X \rightarrow Z$  is pre-Rg-continuous then  $g$  is strongly -rg-continuous.

**Proof :** Let  $V$  be an arbitrary open set in  $Z$ . Since  $g \circ f$  is pre-Rg-continuous,  $(g \circ f)^{-1}(V) = f^{-1}(g^{-1}(V))$  is preopen in  $X$ . Since  $f$  is strongly preopen surjective,  $f(f^{-1}(g^{-1}(V))) = g^{-1}(V)$  is open in  $Y$ . Therefore  $g$  is strongly-rg-continuous.

**Lemma 3.28 :** If  $f : X \rightarrow Y$  and  $g : Y \rightarrow Z$  be two functions. If  $f$  is precontinuous,  $\alpha$ -open and  $g$  is pre-Rg –continuous, then  $gof : X \rightarrow Z$  is pre-Rg-continuous .

**Proof:** Let  $V$  be an arbitrary rg-open subset of  $Z$ . Then  $g^{-1}(V)$  is preopen set in  $Y$ , since  $g$  is pre-Rg-continuous. As,  $f$  is precontinuous and  $\alpha$ -open and  $g^{-1}(V)$  is preopen,  $f^{-1}(g^{-1}(V)) = (gof)^{-1}(V)$  is preopen in  $X$  . This shows that  $gof$  is pre-Rg-continuous.

**Lemma 3.28:** If  $f : X \rightarrow Y$  is surjective M-preopen and  $g : Y \rightarrow Z$  is a function such that  $gof : X \rightarrow Z$  is pre-Rg-continuous then  $g$  is pre-Rg-continuous.

**Proof :** Let  $U$  be an rg-open set in  $Z$ . Since  $gof$  is pre-Rg-continuous,  $(gof)^{-1}(U) = f^{-1}(g^{-1}(U))$  is preopen in  $X$ . Since  $f$  is surjective M-preopen, if  $f^{-1}(g^{-1}(U)) = g^{-1}(U)$  is preopen in  $Y$ . Therefore,  $g$  is pre- Rg-continuous.

**Theorem 3.29:** If  $f : X \rightarrow Y$  is rg-irresolute and  $g : Y \rightarrow Z$  be g-rg-continuous, then  $gof : X \rightarrow Z$  is g-rg-continuous.

**Proof:** Let  $V$  be an arbitrary g-open subset of  $Z$ . Then  $g^{-1}(V)$  is rg-open set in  $Y$ , Since  $g$  is g- rg-continuous. As,  $f$  is rg-irresolute and  $g^{-1}(V)$  is rg-open,  $f^{-1}(g^{-1}(V)) = (gof)^{-1}(V)$  is rg- open in  $X$ . This shoes that  $gof$  is g-rg-continuous.

**Theorem 3.30 :** If  $f : X \rightarrow Y$  be strongly g-continuous with  $Y$  be  $T_{rg}$ space and  $g : Y \rightarrow Z$  be g- rg-continuous, then  $gof : X \rightarrow Z$  is strongly g-continuous.

**Proof :** Let  $V \subset Z$  be an arbitrary g-open set. Since  $g$  is g-rg-continuous, then  $g^{-1}(V)$  is rg-open set in  $Y$ . Given that  $Y$  is  $T_{rg}$ space ,  $g^{-1}(V)$  is g-open in  $Y$  and  $f$  is strongly g-continuous,  $f^{-1}(g^{-1}(V)) = (gof)^{-1}(V)$  is open in  $X$ . This shows that  $gof$  is strongly g-continuous.

**Theorem 3.31 :** If  $f : X \rightarrow Y$  be pre-Rg-continuous and  $g : Y \rightarrow Z$  be pre-rg-continuous, then  $gof : X \rightarrow Z$  is almost precontinuous.

**Proof :** Let  $V$  be an arbitrary regular-open set of  $Z$ . Since  $g$  is pre-rg-continuous, then  $g^{-1}(V)$  is rg-open set in  $Y$ , As  $f$  is pre-Rg-continuous function and  $g^{-1}(V)$  is rg-open in  $Y$ . Then,  $f^{-1}(g^{-1}(V)) = (gof)^{-1}(V)$  is preopen in  $X$ . This shows that  $gof$  is almost precontinuous.

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